

TA comments: Problems 1&2: Nicely done! A handy rule of thumb is that you get about 1 MW-day of energy per gram of fissioned material. Problem 3: Very nicely done!! My numbers came out very similarly. Problem 4: Good discussion. Problem 5: (-0.5) This is a nice discussion of some differences between a PWR and BWR, but the question was specifically asking about what causes the difference in power density between them, not just general differences between them. You touch on this briefly when you talk about transients, though both PWR and BWR will experience this effect under sudden increased thermal load. The idea here is that under regular operation a BWR generates steam in the core, meaning the moderator becomes significantly less dense towards the top of the core. As a result, the average moderating power is reduced, and more moderator by volume is required to maintain the same overall power level. Problem 6: Looks pretty good! Your RI came out a little high, and P_esc a little low, but not too far off. Problem 7: Nicely done!

MIT 22.13 Problem Set 1

Problem 1

Part a

Heat of combustion of methane: $55.5 \cdot 10^6 \frac{J}{kg}$.

Energy of 3 GWth reactor in 24 hrs: $3 \cdot 10^9 \frac{J}{s} \cdot 24 \frac{hr}{day} \cdot 60 \frac{min}{hr} \cdot 60 \frac{s}{min} = 2.592 \cdot 10^{14} J$

Mass of Methane: $\frac{2.592 \cdot 10^{14} J}{55.5 \cdot 10^6 \frac{J}{kg}} = 4.6703 \cdot 10^6 kg$

Part b

Average available energy per U-235 Fission: $\sim 200 \text{ MeV} = 3.2044 \cdot 10^{-11}$

Mass of U-235 atom: $235.044 \text{ amu} = 3.903 \cdot 10^{-25} kg$

Energy per mass of U-235 fission: $\frac{3.0441 \cdot 10^{-11} J}{3.903 \cdot 10^{-25} kg} = 8.21 \cdot 10^{13} \frac{J}{kg}$

Mass of U-235 required: $\frac{2.592 \cdot 10^{14} J}{8.21 \cdot 10^{13} \frac{J}{kg}} = 3.16 \text{ kg}$

Part c

DT reaction energy: $17.59 \text{ MeV} = 2.82 \cdot 10^{-12} J$

Mass of D + T: $2.014102 + 3.0160492 = 5.0302 \text{ amu} = 8.353 \cdot 10^{-27} kg$

Energy per mass of DT: $\frac{2.82 \cdot 10^{-12} J}{8.353 \cdot 10^{-27} kg} = 3.374 \cdot 10^{14} \frac{J}{kg}$

Mass of D-T fuel required: $\frac{2.592 \cdot 10^{14} J}{3.374 \cdot 10^{14} \frac{J}{kg}} = 0.768 \text{ kg}$

Question 2

Mass of the fission products will be equal to the mass of the fuel subtracted by the mass lost to energy ($E=mc^2$). Assume fuel is just U-235. Assume 100% of the energy from fission goes to thermal.

Energy of 1000 MWth reactor in 1 year (90% capacity): $1000 \cdot 10^6 \frac{J}{s} \cdot 365 \cdot 24 \frac{hr}{day} \cdot$

$60 \frac{min}{hr} \cdot 60 \frac{s}{min} \cdot 0.9 = 2.84 \cdot 10^{16} J$

From 1b:

Energy per mass of U-235 fission: $8.21 \cdot 10^{13} \frac{J}{kg}$ (From part 1)

$$\text{Mass of U-235: } \frac{2.84 \cdot 10^{16} J}{8.21 \cdot 10^{13} \frac{J}{kg}} = 345.7 \text{ kg}$$

$$\text{Mass lost to energy: } \frac{2.84 \cdot 10^{16} J}{c^2} = 0.316 \text{ kg}$$

$$\text{Mass of Fission Products: } 363.9 - 0.316 \text{ kg} = 345.4 \text{ kg}$$

Problem 3

Derived in lecture 3, the average logarithmic energy decrement for isotropic scattering:

$$\xi = \left(\ln \frac{E_0}{E} \right) = 1 + \frac{\alpha}{1 - \alpha} \ln \alpha$$

$$\alpha = \left(\frac{A - 1}{A + 1} \right)^2$$

For a molecule $X_a Y_b$:

$$\bar{\xi} = \frac{\xi_x a \sigma_{sx} + \xi_y b \sigma_{sy}}{a \sigma_{sx} + b \sigma_{sy}}$$

Example for UO2:

$$\xi_{UO2} = \frac{\xi_H 2\sigma_{SH} + \xi_{SO} 1\sigma_{SO}}{2\sigma_{SH} + 1\sigma_{SO}}$$

Where the cross sections is taken to be constant (at the plateau).

The total scattering cross-section is the combination of elastic and inelastic cross-sections, the weighting in the above equation must be performed for both:

$$\sigma_s = \sigma_{se} + \sigma_{si}$$

And thus the number of collisions can be approximated as:

$$n \cong \frac{1}{\bar{\xi}} \ln \frac{E_0}{E_n}$$

$$E_0 = 2 \text{ MeV and } E_n = 0.1 \text{ eV}$$

$$\sigma_{SU} = 9.0 \text{ barns}$$

$$\sigma_{SO} = 3.7 \text{ barns}$$

$$\sigma_{SH} = 18.7 \text{ barns}$$

$$\sigma_{SD} = 3.3 \text{ barns}$$

$$\sigma_{SC} = 4.5 \text{ barns}$$

$$\sigma_{SB} = 2.13 \text{ barns}$$

Species	A	alpha	xi	n
H2O	-	-	0.92	18.3
H	1	0	1	16.8
D2O	-	-	0.684	24.6
C	12	0.715976	0.158	106.4
UO2	-	-	0.0382	440.3
B	10	0.669421	0.1873	89.8

Calculate the macroscopic slowing down power:

$$MSDP = \xi \Sigma_S$$

Where $\Sigma_S = \frac{\rho_s N_A \sigma_s}{M}$

Approximate total scattering cross section of the molecule as the total cross-section of the individual atoms making up the molecule:

$$\overline{\sigma_{s,H2O}} = 2 \cdot \overline{\sigma_{s,H}} + \overline{\sigma_{s,O}} = 16.73 \text{ barns}$$

$$\overline{\sigma_{s,D2O}} = 4.6 \text{ barns}$$

$$\overline{\sigma_{s,UO2}} = 23.08 \text{ barns}$$

$$1 \text{ barn} = 1 \cdot 10^{-24} \text{ cm}^2$$

$$\rho_{H2O} \cong 1 \text{ g/cm}^3$$

$$\rho_{D2O} \cong 1.1 \text{ g/cm}^3$$

$$\rho_H \cong 0.0899 \text{ g/cm}^3 \text{ (hydrogen at } 0^\circ \text{C)}$$

$$\rho_C \cong 2.267 \text{ g/cm}^3$$

$$\rho_B \cong 2.08 \text{ g/cm}^3$$

$$\rho_{UO2} \cong 10.97 \frac{\text{g}}{\text{cm}^3} \text{ (Natural Uranium)}$$

Thus, the macroscopic slowing down power of each molecule:

	H2O	D2O	UO2	H (hydrogen Gas)	C	B
σ_s	41.200	10.300	27.700	18.700	4.500	2.130
Density	1.000	1.110	10.970	0.090	2.267	2.080
M	18.000	20.000	270.000	1.000	12.000	10.000
Σ_S	1.378	0.344	0.678	1.012	0.512	0.267
MSDP	1.269	0.235	0.026	445.720	0.081	0.050

The moderating ratio is given by:

$$MR = \frac{\xi \Sigma_S}{\Sigma_a}$$

Use the NNDC data to determine the absorption cross-section. The 3 main reactions are radiative capture (gamma emission), particle ejection (such as alpha), and fission. Using 1/E flux to condense the cross section:

$$\overline{\sigma_a} = \frac{\int \phi \sigma_a(E) dE}{\int \phi dE} = \frac{\sum_{E=0.1eV}^{E=2 \text{ MeV}} \left[\left(\frac{1}{E} \right) \sigma_a(E) \right] \Delta E}{\ln \frac{2 \cdot 10^6}{0.1}}$$

For example, calculate cross-sections for Hydrogen:

For absorption with hydrogen, the only relevant absorption is radiative capture (n,γ), performing the above calculation for flux averaged cross section.

Perform the numerical integration above by using a simple rectangular middle Riemann sum on the point-wise data from NNDC from E=0.1 eV to 2 MeV (Note that the step size ΔE_i is variable as given by NNDC data). Where required, the step sizes are interpolated and discretized evenly):

$$\sum_{E_i=0.1eV}^{E_i=2\text{ MeV}-\Delta E_i} \left[\left(\frac{1}{E} \right) \sigma_a(E) \right] \Delta E = \sum_{E_i=0.1eV}^{E_{i+1}=2\text{ MeV}} f(E^*) \Delta E_i$$

$$E^* = \frac{1}{2} (E_i + E_{i+1})$$

$$\Delta E_i = E_{i+1} - E_i$$

$$f(E^*) = \frac{1}{2} [f(E_i) + f(E_{i+1})]$$

Thus, calculating σ_a :

$$\overline{\sigma_{aH}} = 0.01977 \text{ barn}$$

For oxygen, the only relevant absorption at energies between 0.1 eV and 2 MeV is radiative capture again:

$$\overline{\sigma_{aO}} = 3.61 \cdot 10^{-5} \text{ barn}$$

$$\overline{\sigma_{aH2O}} = \overline{\sigma_{aO}} + 2 \cdot \overline{\sigma_{aH}} = 0.396 \text{ barn}$$

$$MR_{H2O} = \frac{\xi_{H2O} \cdot \Sigma_{sH2O}}{\Sigma_{aH2O}} = \frac{\xi_{H2O} \cdot \sigma_{sH2O}}{\sigma_{aH2O}} = 388.12$$

Performing the same analysis for all species:

	H2O	D2O	UO2	H	C	B
$\sigma_a(\text{barn})$	0.04	$1.98 \cdot 10^{-4}$	16.46	0.02	0.00	228.70
$\sigma_s(\text{barn})$	41.20	10.30	16.4	18.70	4.50	2.13
ξ	0.92	0.68	0.04	1.00	0.16	0.19
MR	958.65	71921.86	0.04	945.97	3023.12	0.0017

Problem 4

Heavy Water Reactor (ie. CANDU):

Advantages:

- Natural uranium resource utilization due to higher neutron economy (results from low adsorption cross section and subsequently a higher (than LWR) moderating ratio).
- On-line refueling is possible due to low excess reactivity of natural uranium. Results in an increased capacity factor
- Fuel fabrication is easier and is produced in simpler units. Further there are more suppliers and sources due to natural uranium use.

Disadvantages:

- Higher plutonium production since natural uranium is used. This is a political/proliferation concern.
- Lower burnup due to lower enrichment, resulting in larger reactor
- Heavy water is expensive
- Tritium generation radioactivity, requires handling and storage. Greater release risk.
- Higher safe-guarding requirement due to on-line refueling capability

Light Water Reactor

Advantages:

- High moderating power due to high neutron energy loss scattering off of hydrogen

- Well researched and commercially available design
- PWR is compact and has a high power density
- Light water is cheap

Disadvantages:

- Requires higher enrichment since water is a higher neutron absorber compared to graphite
- Requires high flow rate of water, increasing costs of the equipment
- Large volume requirement (large pressure containment in BWR)
- High pressures (especially for PWR) in order to achieve higher temperatures for efficiency. This increases the costs for containment design and increases consequence of LOCA.

Fluoride High-Temperature Salt-Cooled Reactor

Advantages:

- High temperature operation results in a high carnot efficiency
- Atmospheric pressure results in potentially reduced cost on pressure containment and potentially lower consequence in LOCA event
- Brayton cycle can be coupled with a gas turbine as a topping cycle to improve overall efficiency

Disadvantages:

- Tritium generation from using lithium salts (required to lower boiling point)
- High corrosion
- Novel design which is not adequately tested. Many uncertainties
- Higher degree of system complexity as control of salt chemistry is required

Problem 5

- lower flow rate than a PWR since heat from vaporization is larger than heat from temperature change. $Q = \dot{m}C_p\Delta T$ vs. $Q = \dot{m}\Delta H_v$
- lower pressure operation
- core receives less irradiation than a PWR, easier for materials design
- operates at a lower fuel temperature (higher safety margin)
- Simplified system due to fewer components required.
- Lower failure/rupture probability due to fewer parts, no steam generator tubes.
- More self-limiting reaction as rate slows down as density decreases. $R = \phi N\sigma$ where N is the atomic density. In event of temperature transient, bubble formation (vaporization) will reduce moderator density. This also means less poisons need to be added resulting in higher breeding of U-238.
- Lower power density means that natural circulation can be used if plant power is lost.

Problem 6

Calculate the potential scattering for Li and F:

$$\begin{aligned}r_0 &= 1.25 \cdot 10^{-13} \text{ cm} \\ R &= r_0 A^{1/3} \\ \sigma_x &= 4\pi R^2 \\ \sigma_{pLi-7} &= 7.185 \cdot 10^{-25} \text{ cm}^2 \\ \sigma_{pF-19} &= 1.398 \cdot 10^{-24} \text{ cm}^2 \\ \sigma_{pU-238} &= 7.51 \cdot 10^{-24} \text{ cm}^2\end{aligned}$$

The atomic density is given by the question:

$$N_{Li} = 0.5 \frac{at}{bcm} = 5 * 10^{23} \frac{atoms}{cm^3}$$

$$N_F = 0.6 \frac{at}{bcm} = 6 * 10^{23} \frac{atoms}{cm^3}$$

$$N_U = 0.1 \frac{at}{bcm} = 1 * 10^{23} \frac{atoms}{cm^3}$$

Calculate the resonance integral using the narrow resonance model:

$$RI_{eff,NR} = \int_{E_{min}}^{E_{max}} \frac{dE}{E} \sigma_{a0}(E) \frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(E) + \sigma_d} = \sum_{E=E_F}^{E=E_0} \frac{\Delta E}{E} \sigma_{a0}(E) \frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(E) + \sigma_d}$$

The integral above is performed numerically again using a rectangular middle Riemann Sum (same as done in question 3).

Where σ_{p0} is the potential cross section of U-238

$$\sigma_{p0} = \sigma_{p-U238} = 7.51 * 10^{-24}$$

σ_d is the dilution cross section defined as follows:

$$\sigma_d = \frac{(\Sigma_{Li7} + \Sigma_{F19})}{N_{U238}} = \frac{N_{Li7}\sigma_{Li7} + N_{F19}\sigma_{F19}}{N_{U238}} = 1.198 * 10^{-23} cm^2$$

Take the absorption cross section of σ_{a0} as the absorption cross section (n,γ) of U-238 between $E_0=0.02$ MeV to $E_F= 4$ eV (resonance range for U-238). Thus, the effective resonance integral (narrow range) is calculated to be:

$$RI_{eff,NR} = 1.68 * 10^{-23} cm^2$$

Thus, the resonance escape probability:

$$p = e^{-\frac{RI_{eff}}{\bar{\xi}\sigma_d}}$$

$$\bar{\xi}_{Li} = 0.26$$

$$\bar{\xi}_F = 0.102$$

$$\bar{\xi} = \left(\ln \frac{E_0}{E} \right) = 1 + \frac{\alpha}{1-\alpha} \ln \alpha$$

$$\alpha = \left(\frac{A-1}{A+1} \right)^2$$

The $\bar{\xi}$ is calculated by weighting by the relative cross sections in the moderator:

$$\bar{\xi} = \frac{\bar{\xi}_{Li}N_{Li}\sigma_{sLi} + \bar{\xi}_FN_F\sigma_{sF}}{N_{Li}\sigma_{sLi} + N_F\sigma_{sF}} = 0.149$$

$$p = e^{-\frac{RI_{eff}}{\bar{\xi}\sigma_d} = 8.21 * 10^{-5}}$$

Problem 7

a)

U-238: 99.28% of natural uranium

U-235: 0.72% of natural uranium

$$\frac{0.72}{99.28} = \frac{N_{235}}{N_{238}} = \frac{N_{U235}^0}{N_{U238}^0} \cdot \frac{e^{-\lambda_{235}t}}{e^{-\lambda_{238}t}} \quad (1)$$

Where $t = 4.5$ billion years

Find the decay constants given half life:

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

$$\lambda_{U235} = 9.7216 \cdot 10^{-10} \text{ years}^{-1}$$

$$\lambda_{U238} = 1.537 \cdot 10^{-10} \text{ years}^{-1}$$

From equation (1) above calculate the fraction of U-235 to U-238 (Mole fraction)

$$\frac{N_{U235}^0}{N_{U238}^0} = 0.2884$$

Thus, when the earth began, the natural enrichment (by mole fraction):

$$\%U235 = \frac{0.2884}{0.2884 + 1} \cdot 100\% = 22.4\%$$

b)

With an enrichment of 4%, calculate the ratio of U235 to U238:

$$\%U235 = \frac{U_{ratio}}{U_{ratio} + 1} = 0.04$$

$$U_{ratio} = 0.0417$$

Solve for time:

$$0.0417 = \frac{N_{U235}^0}{N_{U238}^0} \cdot \frac{e^{-\lambda_{235}t}}{e^{-\lambda_{238}t}} = 0.2884 * e^{(-\lambda_{235} + \lambda_{238})t}$$

$$t_{4\%} = 2.36 \text{ billion years}$$

The enrichment on earth was 4% when the earth was 2.36 billion years old.

The enrichment of U-235 was 4% 2.14 billion years ago.